INSPIRE NURTURE BELIEVE ACHIEVE

*Working **together** to be the **best** that we can be.*

Maths Calculation Policy and Progression 2022-2023

Happiness	Perserverance	Resilience	Kindness	Friendship	Respect

1.0 Rationale

At the Goldsborough Sicklinghall Federation we believe that Mathematics teaches us how to make sense of the world around us through a child's ability to calculate, to reason and to solve problems. It enables children to understand and appreciate relationships and pattern in both number and space in their everyday lives. We believe that this is underpinned by effective quality teaching and a balanced, motivating curriculum.

1.1 Aims

- To become fluent in the fundamentals of mathematics, including varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately;
- To reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, developing an argument, justification or proof using mathematical language;
- To solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions;
- To promote enjoyment and enthusiasm for learning through practical activity, exploration and discussion;
- To develop in pupils an interest in numbers;
- To understand the importance of mathematics in everyday life.

1.2 Key Principles

The content and principles underpinning the mathematics curriculum reflect those found in high performing education systems internationally, particularly those of east and south-east Asian countries such as Singapore, Japan, South Korea and China.

The principles and features that characterise this 'mastery' approach are:

- Teachers reinforce an expectation that all pupils are capable of achieving high standards in mathematics.
- The large majority of pupils progress through the curriculum content at the same pace. Differentiation is achieved by emphasising deep knowledge and through individual support and intervention.
- Teaching is underpinned by methodical curriculum design and supported by carefully crafted lessons and resources to foster deep conceptual and procedural knowledge.
- Practice and consolidation play a central role. Carefully designed variation within this builds fluency and understanding of underlying mathematical concepts in tandem.
- Teachers use precise questioning in class to test conceptual and procedural knowledge, and assess pupils regularly to identify those requiring intervention so that all pupils keep up.

2.1 Planning

At the Goldsborough Sicklinghall Federation we follow the principles set out in the National Curriculum Framework of study 2014 and the Statutory Framework for the Early Years Foundation Stage.

- Maths is taught on a daily basis from EYFS – Y6;
- Teachers use a variety of teaching methods to deliver the curriculum to achieve end of year national expectations (age related expectations, AREs);
- Teaching and learning takes place largely within a whole class setting
- Same day intervention is used to help children if they need to 'catch up'.
- CPA (Concrete, Pictorial, Abstract) representations are chosen carefully to help build procedural and conceptual knowledge together.
- The long-term curriculum maps allow Maths topics to be taught for longer so children have time to practice and consolidate. There are revision weeks allocated to revisit topics too.
- The Maths Long Term plan focuses upon the White Rose Mixed Age planning scheme V3.0. The LTP breaks down the learning into smaller steps which staff are able to use to plan smaller steps for lessons on the short-term planning format alongside PowerPoint resources.
- In Foundation stage, the emphasis is on teaching Maths through stories and books with strong topic links, child initiated learning through play, with some adult intervention, demonstration and use of the language. In foundation, the mastery approach allows children to gain a secure understanding of number. This is achieved by using the NCETM Mastering Number scheme, alongside White Rose resources. Number Blocks is a key resource and Maths is embedded into routines such as using maths when tidying away.
- Teachers carefully identify the vocabulary that is being taught, useful stem sentences and any possible misconceptions.
- Teachers in EYFS and KS1 have received training for the NCETM Mastering Number scheme and weave this through their practice.

2.2 Mathematics journals and folders

A maths journal (Red book) is used to record maths work and thinking. It is a good tool to help children to articulate their ideas and to record the solutions to maths problems, along with the strategy and thought processes used to arrive at the solution. The children also have a green fluency book to record arithmetic practice and when exploring number relationships and patterns. Tapestry is used to record the work of children in EYFS, alongside the red books.

2.3 Pupil support and differentiation

Taking a mastery approach, differentiation occurs in the support and intervention provided to different pupils, not in the topics taught, particularly at earlier stages. There is no differentiation in content taught but the questioning and scaffolding individual pupils receive in class as they work through problems will differ, with higher attainers challenged through more demanding problems which deepen their knowledge of the same content. Pupils' difficulties and misconceptions are identified through immediate formative assessment and addressed with same day intervention. This may take place alongside the teacher within the classroom or through targeted sessions in a smaller group setting or through interventions. Teachers also use the 'NCETM Mastery Assessment' materials as tasks, questions and activities to teach the same curriculum content to the whole class, challenging the rapid graspers by supporting them to go deeper rather than accelerating some pupils into new content.

3.1 Curriculum content and curriculum design

A detailed, structured curriculum is mapped out across all year groups, ensuring continuity and supporting transition. Objectives are in relatively small carefully sequenced steps, which must each be mastered before pupils move to the next stage. Fundamental skills and knowledge are secured first. This often entails focusing on curriculum content in considerable depth at early stages. We base our maths lessons on the 'White Rose Mixed Age SOL v 3.0' and the 'NCETM: Mastering Number' programme in EYFS.

3.2 An Example of our lessons: Pupils are set a fluency task to develop their mental recall and knowledge of number facts, this will often lead into the main objective for the lesson. Pupils are introduced to the main lesson content and how it relates to real life, any new vocabulary is then introduced, with teachers insisting it is used throughout the lesson. Examples of the new concepts are designed to prompt discussion and reasoning. These are often presented with objects (concrete manipulatives) for children to use as it is important that children move through the concrete, pictorial, abstract representations. Teachers also use careful questions to draw out pupils' discussions and their reasoning. The class then try some questions in 'Guided Practice'. Carefully designed variation in these questions builds fluency and deep understanding. When they are ready to apply their learning independently, the children answer questions in their journal or on scaffolded worksheets. If some children are not ready by this point, they will continue 'Guided Practice' with the teacher or TA in a small group. As the children complete their independent tasks, teachers will give immediate feedback though marking as they go along or through peer marking. This is an effective way to quickly identify misconceptions and close the gap. Children in EYFS will follow the NCETM 'Mastering Number' scheme Monday to Thursday, whilst the White Rose Maths content will taught explicitly on a Friday, as well as being weaved through the children's daily continuous provision.

4.1 Resources

A range of high quality curriculum materials are used to support classroom teaching. Concrete and pictorial representations of mathematics are chosen carefully to help build procedural and conceptual knowledge together. Exercises are structured with great care to build deep conceptual knowledge alongside developing procedural fluency. The focus is on the development of deep structural knowledge and the ability to make connections. Making connections in mathematics deepens knowledge of concepts and procedures ensures what is learnt is sustained over time and cuts down the time required to assimilate and master later concepts and techniques. The Goldsborough Sicklinghall Federation follows the White Rose guidance as the programme encourages extensive practice to develop fluency and mastery so that every child, across all abilities, can succeed in mathematics. The use of Mathematics resources is integral to the concrete – pictorial – abstract approach and thus planned into our learning and teaching. Teachers' resources are largely based on the White Rose Materials and NCETM Professional Development Materials.

4.2 Times Table Rock Stars

At the Goldsborough Sicklinghall Federation we ensure that the children have plenty of opportunities to develop their quick recall of key number bonds, facts and times tables, as this helps the children to become more confident when solving problems and calculations.

The Goldsborough Sicklinghall Federation has signed up for Time Tables Rockstars to help support the children in learning, practicing and consolidating their recall of multiplication and division facts.

TT Rock Stars is a carefully sequenced programme of times tables practice. Teachers choose appropriate times tables for their class and this is set as homework to accompany the time table work carried out in class. Our aim is to improve confidence and speed in times tables to help them with more complex maths skills. In Year 1, the children complete similar activities but with a focus on number bonds and addition and subtraction facts using NUMBOTS.

4.3 Homework

At Goldsborough school children are given maths homework every other week. The aim of this exercise is for children to strengthen their knowledge of key facts and to share their learning with their parent or carer.

5.1 Maths Non-negotiables

Each class from Reception to Y6 has a set of Maths Non-Negotiables, which we refer to as KIRFs (Key Instant Recall Facts). These are generally the essential elements and 'basics' of Maths that are crucial for children's mathematical learning and progression. Of course, all areas of Maths are important and valuable but a focus on the 'basics' can help to advance and secure their learning across this subject.

6.0 Assessment

- Progress in mathematics will be monitored through ongoing teacher assessments and children's work in maths is marked in accordance with the school's Marking and Feedback Policy.
- Teachers comment on children's grasp of concepts during a unit on the planning and use this to form their Teacher Assessment and next steps.
- Teachers use the 'Mastery Assessment' materials for tasks, questions and activities to teach the same curriculum content to the whole class but give a range of questions that will also challenge the 'rapid graspers' by supporting them to go deeper rather than accelerating some pupils into new content.
- Progress in times tables is monitored using the TT Rock Star challenges and often weekly quizzes.
- Children in Year 4 undertake a multiplication test in the Summer term.
- The KIRFs provide targets for children.
- In Reception children are assessed against the Development Matters statements and the ELGs that form part of the Foundation Stage Profile;
- Progress in mathematics is monitored using our termly Pupil Progress Meetings. Children's books form the main evidence base for progress and are monitored by termly 'Book Looks' conducted by SLT and Subject Coordinators in collaboration with Teacher conversations. Feedback is given both at individual and whole school level;
- Arithmetic and Maths tests (White Rose) are set at the beginning of each year from Y1 to Y6 to gain a baseline. Staff use this as a basis to review Maths progress and ensure children are appropriately challenged.
- Y2 and Y6 undertake statutory SATs for arithmetic and reasoning at the end of the year. Reports indicate to parents where their achievement sits in relation to the expected standard;
- Children in Years 1, 2, 3, 4, 5 and 6 use the White Rose Assessments each term to highlight progress made and areas of concern for the following term / year.

7.1 Inclusion, SEN & Equal Opportunities

Mathematics lessons, tasks and materials can be differentiated by the class teacher to meet the needs of individual children. Children identified as having Special Educational Needs may need differentiation and support of materials and tasks consistent with that child's stage of learning.

A number of intervention strategies are used to develop children's specific learning needs in Mathematics. These often run for a number of weeks (e.g. a term) and then impact is assessed using entry and exit data. These extra interventions do not take place during core teaching time. These may include the Plus 1, The Power of 2 and our Subitising Intervention.

Adult support is offered to children with SEN regularly, but is not used exclusively in every lesson, so children do not become over-reliant on adult support. These children may use a tailored range of resources based upon the teacher assessment, ideally using B-squared to pinpoint gaps in learning. Children will also be explicitly taught how to use physical resources effectively to aid learning in maths.

(See also our Special Educational Needs Policy)

7.2 Inclusion

All children will be given opportunities to participate on equal terms in all maths activities and due consideration will be given to the principles of Inclusion. Same day intervention is used to prevent gaps in attainment opening up wherever possible.

8.0 Role of subject coordinator: monitoring and evaluation

Monitoring of Standards of children's work and the quality of teaching in mathematics is the responsibility of the mathematics subject coordinator (alongside SLT) in the Goldsborough Sicklinghall Federation. This involves book sampling, pupil interviews, target setting, analysing assessment grids and SATs papers, and informal discussion with colleagues. The work of the subject coordinator and SLT also involves supporting colleagues with the teaching, planning, informing staff about current developments in the subject and providing a strategic lead and direction for the subject in the school. TRG- Maths Teacher Research Groups are continually supporting staff with maths teaching and is an effective collaborative approach which evaluates and develops good practice.

9.0 Policy Review

This policy was agreed by staff and approved by Governors in September 2022. It will be reviewed on a three-year cycle.

Signed: (Headteacher) **Signed:** (Chair of Governors/ sub-committee)

Date: 04.1.23

The following Goldsborough Sicklinghall Federation Calculation Progression document has been written to support teachers with the National Curriculum and to support our implementation of the Shanghai and Maths Mastery approaches to teaching lessons. We also use the NCETM resource materials to ensure methods are taught in a progressive manner. This involves three spines: Number, addition and subtraction; Multiplication and division; and Fractions.

Key to successful implementation of a school calculation policy is the consistent use of representations (models and images that support conceptual understanding of mathematics) and this policy promotes a range of relevant representations across the primary years. Mathematical understanding is developed through the use of representations that are first of all concrete. (eg. Numicon, Dienes apparatus) and then pictorial (array, place value counters) and then to facilitate abstract working. At the Goldsborough Sicklinghall Federation we use a range of strategies to develop their calculation skills. Here are some of the important key elements to develop children conceptual and procedural fluency with calculations:

1. Develop children's fluency with basic number facts

Fluent computational skills are dependent on accurate and rapid recall of basic number bonds to 20 and times-tables facts. At the Goldsborough Sicklinghall Federation we have found that spending a short time every day on these basic facts quickly leads to improved fluency. One way this is carried out is through our TTRockstar timestable challenges. We are clear that this is not meaningless rote learning; rather, this is an important step to developing conceptual understanding through identifying patterns and relationships between the tables (for example, that the products in the 6× table are double the products in the 3× table). This has helped children develop a strong sense of number relationships, an important prerequisite for procedural fluency. Children in Shanghai learn their multiplication tables in this order to provide opportunities to make connections. At the Goldsborough Sicklinghall Federation we learn our times tables in this order so children can identify patterns and relationships.

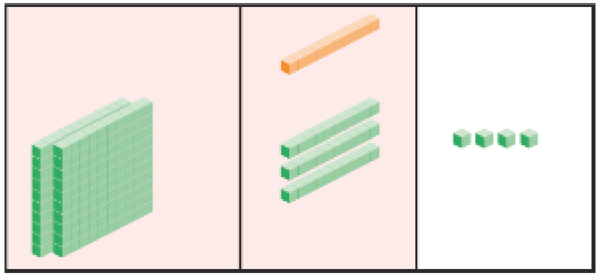
×10	×5	×2	×4	×8	×3	×6	×9	×7

2. Develop children’s fluency with mental and written methods

Efficiency in calculation requires having a variety of mental strategies. It is important that children can mentally recall number bonds and are able to partition numbers in order to bridge through ten. Children are taught that it is helpful to make 10 as this makes the calculation easier.

Teaching column methods for calculation provides the opportunity to develop both procedural and conceptual fluency. We ensure that children understand the structure of the mathematics presented in the algorithms, with a particular emphasis on place value. This is developed through the use of base ten apparatus and the use of models and images in the textbooks to support the development of fluency and understanding. Informal methods of recording calculations are also an important stage to help children develop fluency with formal methods of recording. Informal methods are only used for a short period, to help children understand the internal logic of formal methods of recording calculations. They are stepping stones to formal written methods. For example,

	h	t	o
			8
+	2	3	6
		1	4
		3	0
+	2	0	0
	2	4	4



	h	t	o
			8
+	2	$\overset{1}{3}$	6
	2	4	4

3. Don’t count, calculate

Children benefit from being helped at an early stage to start calculating, rather than relying on ‘counting on’ as a way of calculating. For example, with a sum such as: 4 + 7 = Rather than starting at 4 and counting on 7, children could use their knowledge and bridge to 10 to deduce that because 4 + 6 = 10, so 4 + 7 must equal 11.

4. Look for patterns and make connections

Children are given opportunities in the lessons to look for patterns and make connections. The maths journals are used to explore patterns, relationships and reasoning. The question "What's the same, what's different?" is used frequently to make comparisons.

5. Use of intelligent Practice

The practice children engage in provides the opportunity to develop both procedural and conceptual fluency. Children are required to reason and make connections between calculations. Calculations are chosen carefully to develop children's connections and strategies. Carefully selected calculations provide opportunities for making these connections. For example,

2 × 3 =	6 × 7 =	9 × 8 =
2 × 30 =	6 × 70 =	9 × 80 =
2 × 300 =	6 × 700 =	9 × 800 =
20 × 3 =	60 × 7 =	90 × 8 =
200 × 3 =	600 × 7 =	900 × 8 =

Make 10 and add.

(a) 2 + 8 + 4 = ___ + ___ = ___ (b) 3 + 9 + 1 = ___ + ___ = ___

Add.

(a) 6 + 7 + 4 = ___ (b) 9 + 0 + 4 = ___

(c) 8 + 5 + 9 = ___ (d) 7 + 9 + 6 = ___

6. Move between the concrete and the abstract

Children's conceptual understanding and fluency is strengthened if they experience concrete, visual and abstract representations of a concept during a lesson. Moving between the concrete and the abstract helps children to connect abstract symbols with familiar contexts, thus providing the opportunity to make sense of, and develop fluency in the use of, abstract symbols. Maths lessons at The Goldsborough and Sicklinghall Federation move between the concrete, visual and abstract.

7. Contextualise the maths

The children are posed with contextualised problems. This supports the children's understanding of the abstract calculation.

8. Use of questioning to develop reasoning

Teachers have a strong and consistent focus on questioning that encourages and develops their mathematical reasoning. For example, there is always an emphasis on the 'how do you know?' as opposed to 'what is the answer?' Children know that they need to explain how they worked out a calculation or solved a problem and justify their reasoning.

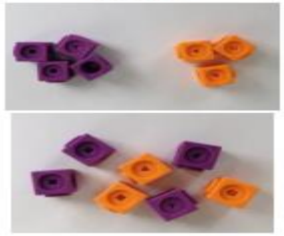
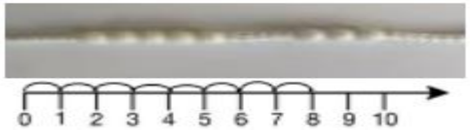
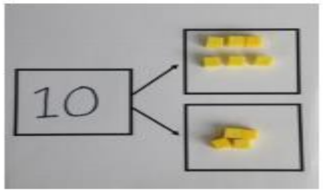
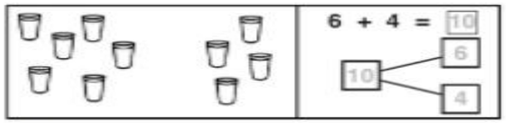
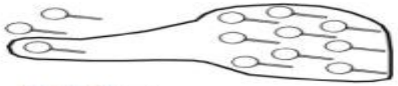
9. Use of precise mathematical vocabulary

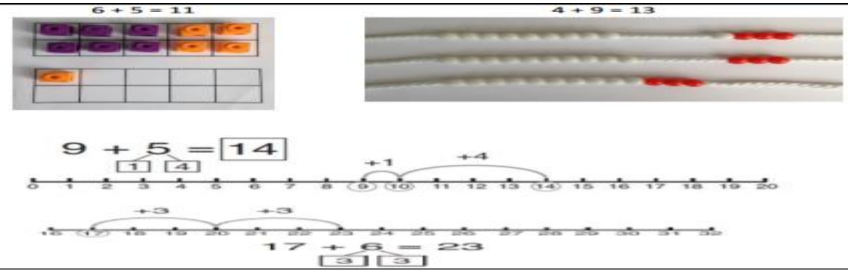
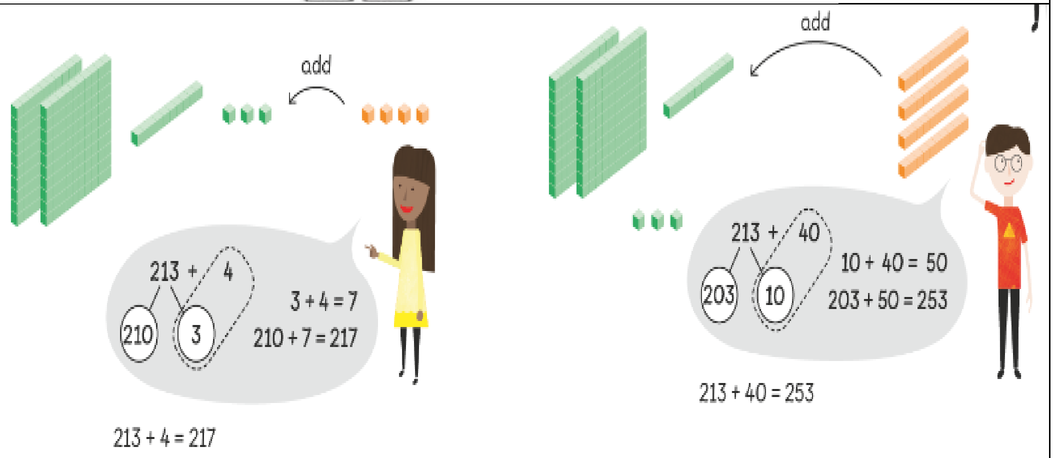
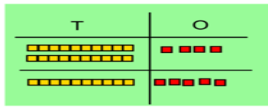
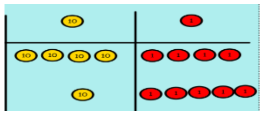
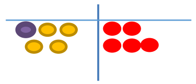
The quality of children's mathematical reasoning and conceptual understanding is significantly enhanced if they are consistently expected to use correct mathematical terminology (e.g. saying 'digit' rather than 'number'). By all using the precise vocabulary, everyone is clear which part of the calculation we are talking about e.g., divisor, dividend, quotient. High expectations of the mathematical language used are essential, with staff only accepting what is correct. Consistency across the school is key. All staff are aware of the correct terminology, as this is located on the Maths Vocabulary Progression document.

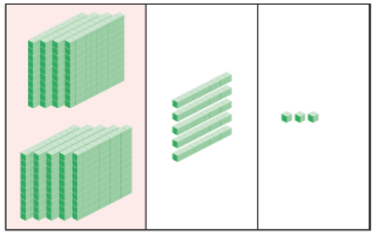
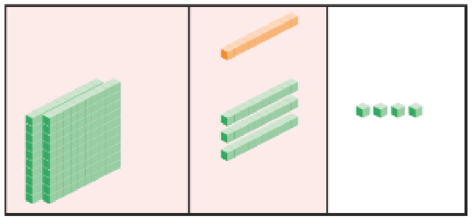
Examples of precise Vocabulary	
Ones	Factor product
is equal to (is the same as)	Whole part whole
Exchange exchanging regrouping	Dividend divisor quotient
calculation equation	Known unknown
bar model	

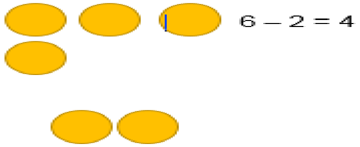
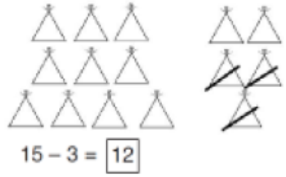
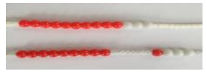
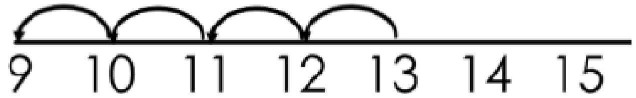
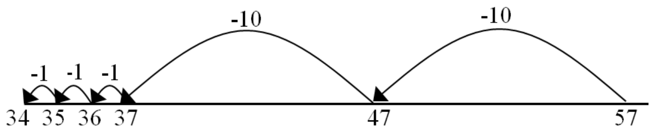
10. Identifying misconceptions

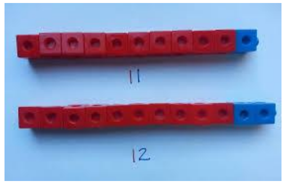
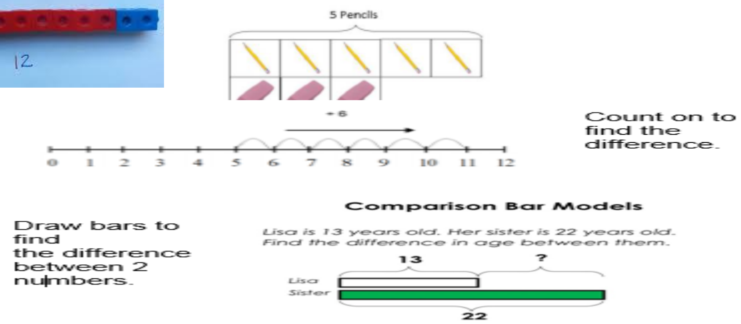
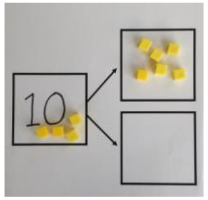
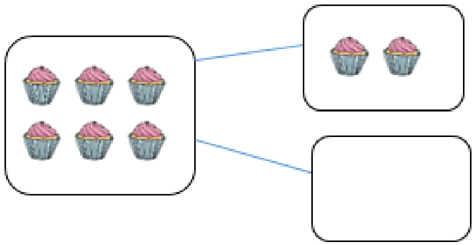
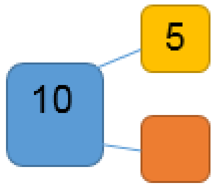
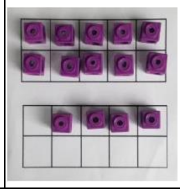
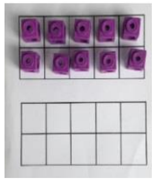
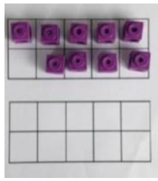
Difficult points need to be identified and anticipated when lessons are being designed and these need to be an explicit part of the teaching, rather than the teacher just responding to children's difficulties if they happen to arise in the lesson. The teacher should be actively seeking to uncover possible difficulties because if one child has a difficulty it is likely that others will have a similar difficulty. Difficult points also give an opportunity to reinforce that we learn most by working on and through ideas with which we are not fully secure or confident. Discussion about difficult points can be stimulated by asking children to share thoughts about their own examples when these show errors arising from insufficient understanding. For example: A visualiser is a valuable resource since it allows the teacher quickly to share a child's thinking with the whole class.

Addition - Strategy and Guidance	**Concrete/Pictorial/Abstract**
Count all 1:1 correspondence Joining two groups and then recounting all objects using 1:1 correspondence	3 + 4 = 7   5 + 3 = 8
Counting on As a strategy, this should be limited to adding small quantities only (1,2 or 3) with pupils understanding that counting on from the greater number is more efficient.	8 + 1 = 9   15 = 12 + 3  
Part whole model Teach both addition and subtraction alongside each other as pupils will use the model to see the inverse relationships between them. This model begins to develop the understanding of commutativity of addition, as pupils will become aware that the parts will make the whole in any order.	  10 = 6 + 4 10 − 6 = 4 10 − 4 = 6 10 = 4 + 6
Regrouping ten ones to make ten. This is an essential skill that will support column addition later on.	 3 + 9 = 3 + 9 = 12 

<u>'Make ten' strategy</u> Pupils should be encouraged to start at the greater number and partition the smaller number to make ten. The ten frames are good for modelling and exploring this.	
<u>Partitioning to add</u>	
<u>Column method no regrouping</u>	24 + 15= Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.  After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.  <u>Examples,</u> 

	Calculations 21 + 42 = 21 + 42  432 + 521 = 953 h t o 4 3 2 + 5 2 1 9 5 3
<u>Expanded column method</u> This is a bridging process before pupils reach the full algorithm for column method. This can be used for struggling learners who are not yet ready to move to the complete column method.	h t o 8 + 2 3 6 1 4 3 0 + 2 0 0 2 4 4
<u>Column method with regrouping</u> This is the standard column method. Show both this and the expanded methods together so pupils can see the link between the two and feel more comfortable using the column method. It is important to go through different methods so that pupils get an understanding of the question and numbers, rather than to just follow a procedure.	Make both numbers with the Base 10. Add up the units and exchange 10 ones for one 10. Add up the rest of the columns. As children move on to decimals, money and decimal place value counters can be used to support learning.  8 + 236 = 244 h t o 8 + 2 $\overset{1}{3}$ 6 2 4 4

Subtraction - Strategy and Guidance	Concrete/Pictorial/Abstract
Count all 1:1 correspondence- Taking away ones	Use physical objects, counters, cubes etc to show how objects can be taken away.  Cross out drawn objects to show what has been taken away. 
Counting back Pupils may start off by counting back but they should be quickly encouraged to rely on number bonds knowledge as time goes on, rather than using counting back as their main strategy.	**Concrete:** Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.  13 – 4 Use counters and move them away from the group as you take them away counting backwards as you go.  **Pictorial:** Count back on a number line or number track  Start at the bigger number and count back the smaller number showing the jumps on the number line.  This can progress all the way to counting back using two 2 digit numbers. **Abstract**: Put 13 in your head, count back 4. What number are you at? Use your fingers to help.
Finding the difference	**Concrete/Pictorial:** Compare amounts and objects to find the difference.

	 Use cubes to build towers or make bars to find the difference Use basic bar models with items to find the difference  **Abstract:** Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.
Part whole model Teach both addition and subtraction alongside each other, as the pupils will use this model to identify the link between them.	 Link to addition- use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 = Use a pictorial representation of objects to show the part part whole model.   Move to using numbers within the part whole model.
'Make ten' strategy	**Concrete:** 14 – 5 =    Make 14 on the ten frame. Take away the 4 first to make 10 and then takeaway 1 more so you have taken away 5. You are left with the answer of 9.

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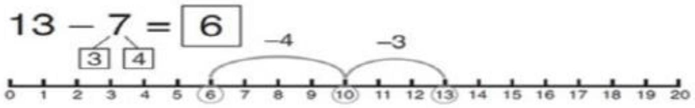
Pictorial:

Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.



Abstract: 16 – 8= How many do we take off to reach the next 10?
How many do we have left to take off?

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<u>Partitioning to subtract</u>
It is important to look at different ways to partition numbers.

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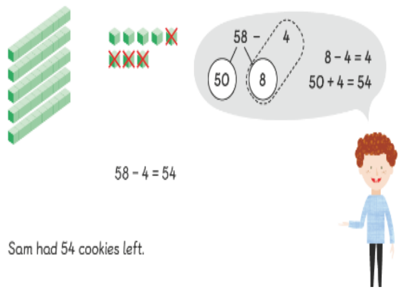
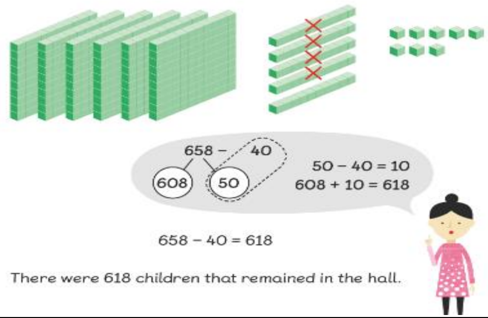
<u>Examples</u>





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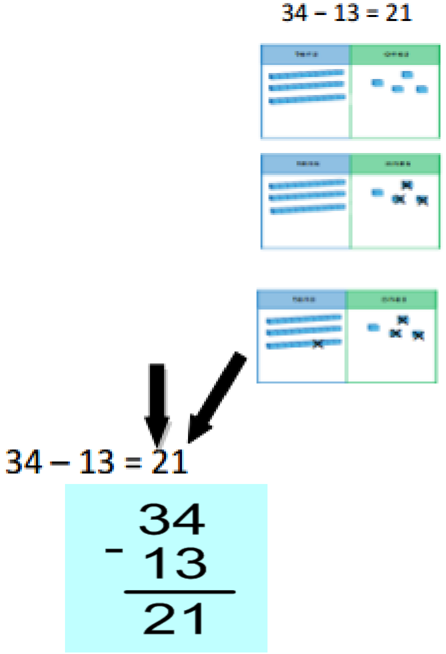
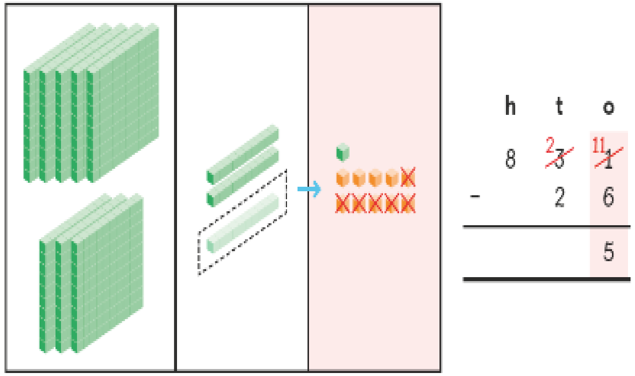
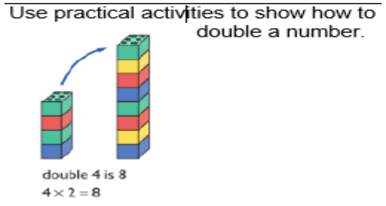
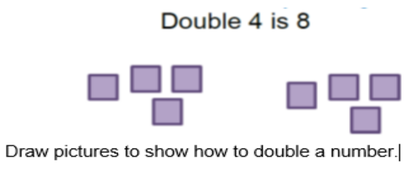
Column method no regrouping Use Base 10 to support their understanding. Misconception: Pupils don't recognise that they are subtracting ones from ones, tens from tens, hundreds from hundreds and inappropriately subtract digits in the wrong columns.	 **Concrete:** Use Base 10 to make the bigger number then take the smaller number away. Show how you partition numbers to subtract. For example, 34 = 30 + 4 13 = 10 + 3 **Pictorial**: Children may wish to draw the Base 10 or place value counters alongside the written calculation to help to show working. **Abstract:** This will lead to a clear written column subtraction.
Column method with regrouping This example shows how pupils should work practically when being introduced to this method. Allow pupils time to explore the written method, showing that they removed a ten and created more ones. It is important here that pupils understand why they are crossing out the tens column and ones column and changing the numbers. Misconception: Pupils don't recognise that they can exchange tens for ones as the tens sticks are stuck together.	Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with two exchanges. 

<table>
<tr><th>Multiplication - Strategy and Guidance</th><th>Concrete/Pictorial/Abstract</th></tr>
<tr><td>Doubling</td><td>

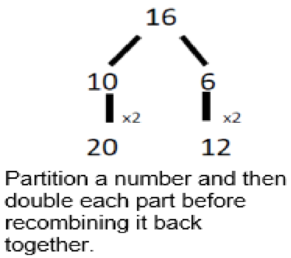
Concrete:



Pictorial:



Abstract:



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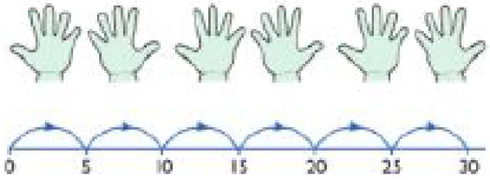
<tr><td>Counting in multiples</td><td>

Concrete: Count in multiples supported by concrete objects in equal groups.

Pictorial: Use a number line or pictures to continue support in counting in multiples.



Abstract: Count in multiples of a number aloud. Write sequences with multiples of numbers. For example, 2, 4, 6, 8, 10

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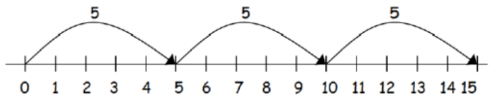
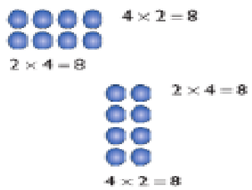
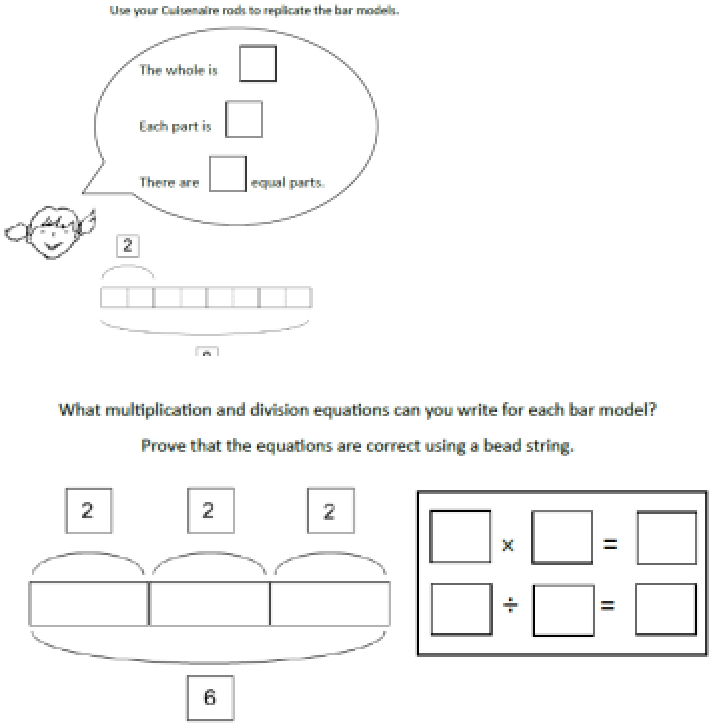
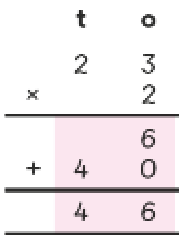
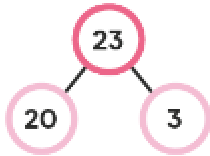
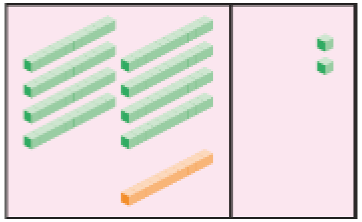
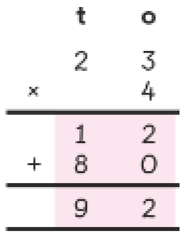
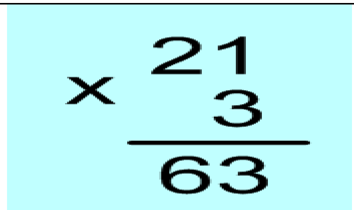
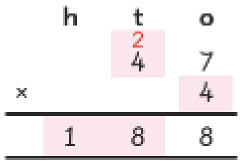
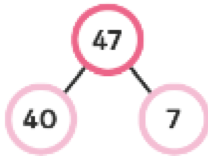
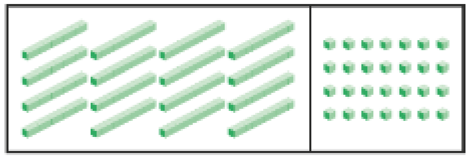
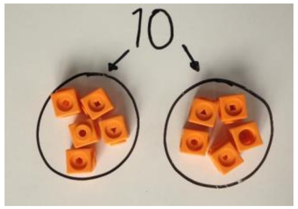
<u>Repeated addition</u>	Concrete:  Pictorial:  Use different objects to add equal groups. There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?  2 add 2 add 2 equals 6  5 + 5 + 5 = 15
<u>Arrays- showing commutative multiplication</u> Pupils should understand that an array and, later, bar models can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.	 **Concrete:** Create arrays using counters / cubes to show multiplication sentences. **Pictorial**: Draw arrays in different rotations to find **commutative** multiplication sentences.   3 × 5 = ☐ 5 × 3 = ☐

	Abstract: Use an array to write multiplication sentences and reinforce repeated addition. 5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 3 x 5 = 15 5 x 3 = 15
<u>Part whole model</u> Use of part-part whole model to establish the inverse relationship between multiplication and division This link should be made explicit from early on, using the language of the part-part-whole model, so that pupils develop an early understanding of the relationship between multiplication and division. Bar models (with Cuisenaire rods) should be used to identify the whole, the size of the parts and the number of parts.	

<u>Expanded Column Multiplication</u> Show this method alongside the partitioning model and the Base 10.	  Step 1 Multiply the ones by 2. 3 ones × 2 = 6 ones Step 2 Multiply the tens by 2. 2 tens × 2 = 4 tens Step 3 Add the products. 6 + 40 = 46 23 × 2 = 46
<u>Expanded Column Method with regrouping.</u>	 
<u>Efficient Column method no regrouping</u>	 Base 10 or place value counters can be used alongside this too.
<u>Efficient Column method with regrouping</u> It is important that the children are able to see all the representations so they can fully understand the concept and not just a procedure. The children need to be using the vocab of how many tens make one hundred, how many ones in a ten etc.	   7 ones x 4 = 28 ones 28 ones = 2 tens and 8 ones 4 tens x 4 = 16 tens 16 tens = 1 hundred and 6 tens

Long Multiplication	32 x 24 8 (4 x 2) 120 (4 x 30) 40 (20 x 2) 600 (20 x 30) 768	Start with long multiplication, reminding the children about lining up their numbers clearly in columns. If it helps, children can write out what they are solving next to their answer.

Division - Strategy and Guidance	Concrete/Pictorial/Abstract
Sharing objects into groups	**Concrete:** I have 10 cubes, can you share them equally in 2 groups?  

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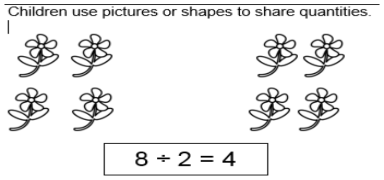
Pictorial:



Abstract: Share 9 buns between three people.9 ÷ 3 = 3

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<u>Division as grouping</u>

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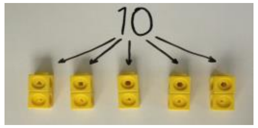
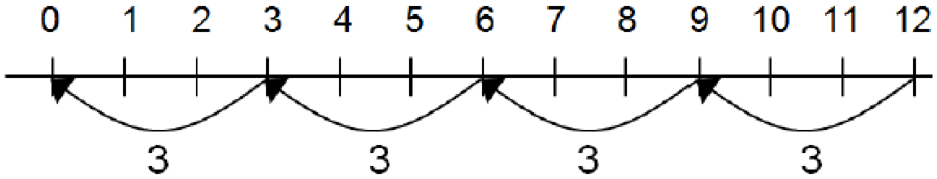
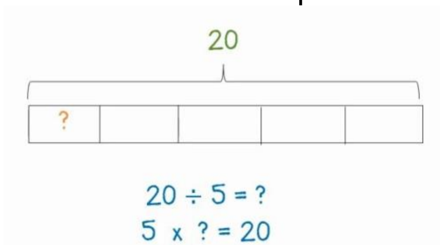
Concrete: Divide quantities into equal groups.
Use cubes, counters, objects or place value counters to aid understanding.



Pictorial: Use a number line to show jumps in groups. The number of jumps equals the number of groups.



Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.
Use a number line or pictures to continue support in counting in multiples.



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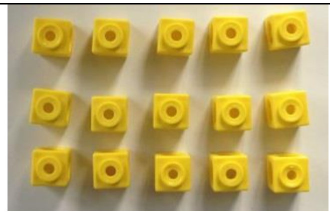
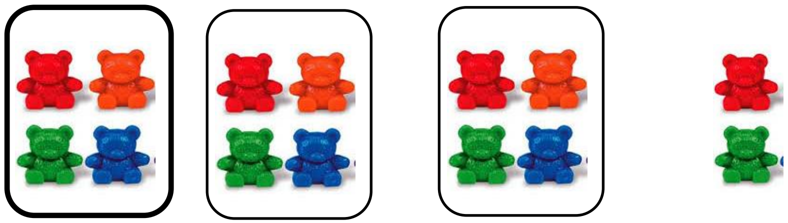
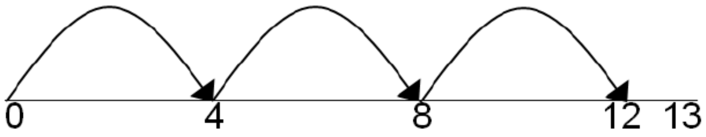
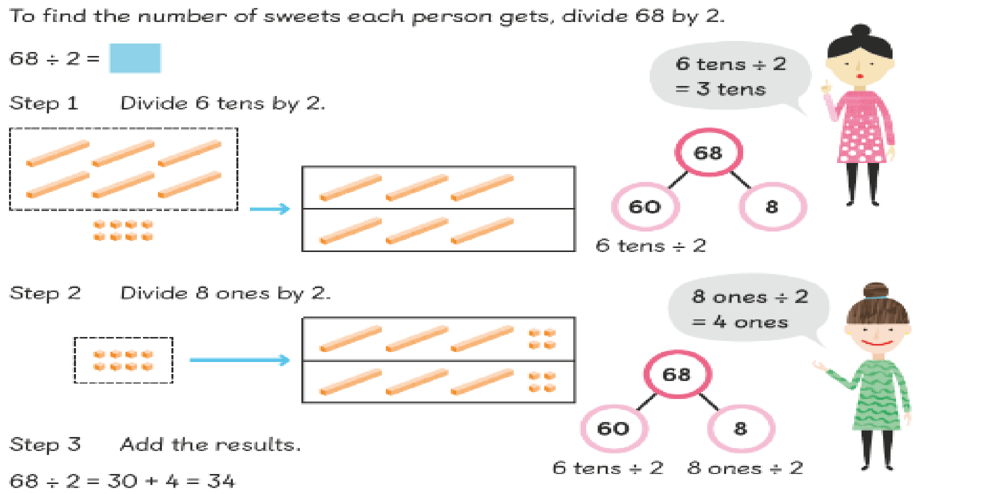
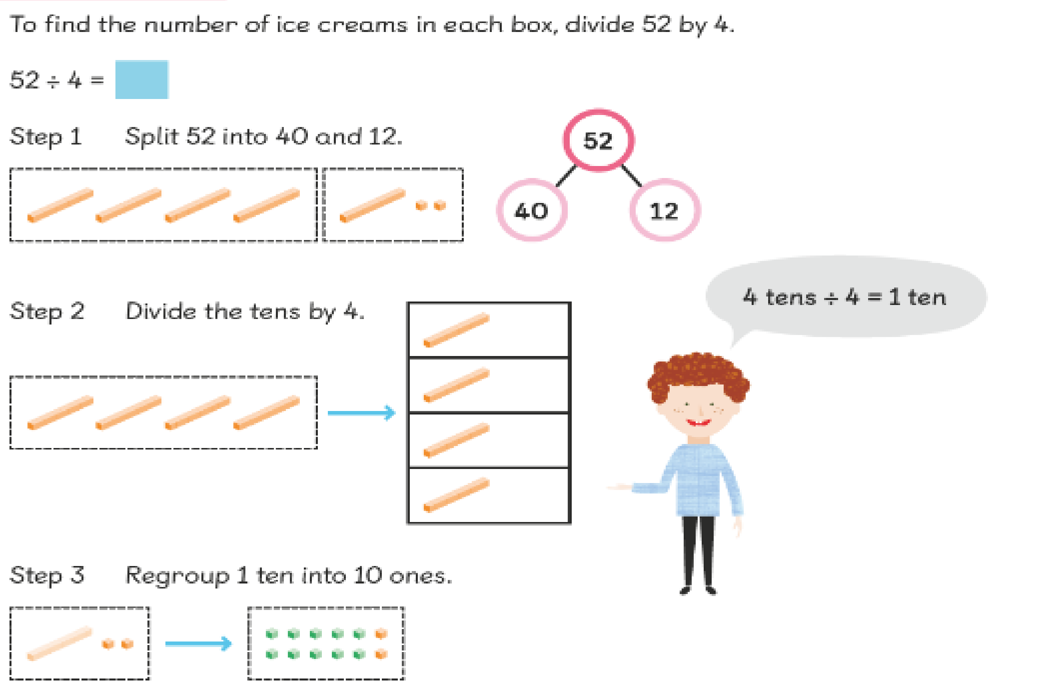
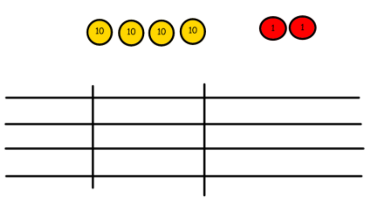
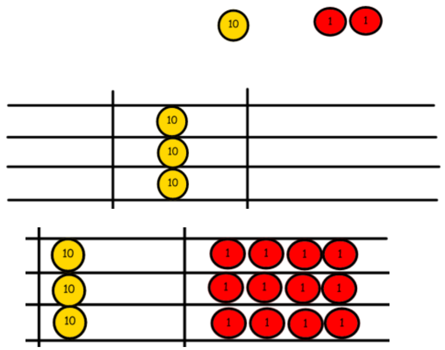
	Abstract: 28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?
<u>Division within arrays</u>	 Concrete: Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15 Pictorial: Draw an array and use lines to split the array into groups to make multiplication and division sentences. Abstract: Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7
<u>Division with remainders</u>	**Concrete:** 14 ÷ 3 = Divide objects between groups and see how much is left over 

	Pictorial: Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder.  **Abstract:** Complete written divisions and show the remainder using r. 
<u>Partitioning</u>	 Partitioning with regrouping

	
<u>Expanded Division Method</u>	1 2 8) 9 6 − 8 0 1 6 − 1 6 0
<u>Short Division</u>	 Calculations 42 ÷ 3 42 ÷ 3= **Concrete/pictorial**: Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.



We exchange this ten for ten ones and then share the ones equally among the groups. We look at how much in 1 group so the answer is 14.

Abstract: Begin with divisions that divide equally with no remainder.
Move onto divisions with a remainder.
Finally move into decimal places to divide the total accurately.

```
      8 6  r 2
   ┌────────
   │    3
 5 │ 4 3 2
```

```
     2 1 8
   ┌──────
   │     3
 4 │ 8 7 2
```

```
          1  4  .  6
      ┌──────────────
      │      16    21
 3 5  │ 5 1  1  .  0
```